Closing Wed: HW_9A,9B (9.3/4,3.8)
Final: Sat, June 3th, 1:30-4:20, ARC 147

New material for the final, be able to:
Solve separable diff. eq..
Use initial conditions & constants.
Be able to set up the applied problems from homework.
Worried about applied problems?
Go thru my review sheets and
look at old finals.

Newton's Cooling Law Experiment

Hot water is in the cup. We will try to predict the temp. at the end of class.

1st measurement:

Time = Temp =

2nd measurement:

Time = Temp =

9.4 Differential Equations Apps

***1. Law of Natural Growth/Decay*:**
Assumption: "*The rate of growth/decay is proportional to the function value*."

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

We solve this in general last time and got

$$P(t) = P_0 e^{kt}$$

Examples:
a. A pop. has 500 bacteria at t=0. After 3 hrs there are 8000 bacteria. Assume the pop. grows at a rate proportional to its size.
Find B(t).

b. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. Find m(t).

c. Bob deposits $2000 into a savings account. The money grows at a rate proportional to its size (i.e. compound interest like almost all bank account). The balance in 4 years is $2100. Find the formula B(t) for the amount in his account in t years.

2. Newton’s Law of Cooling:

Assumption: “*The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings*.”

3. Mixing Problems:

Assume you have a vat of liquid that has a substance (a contaminant) entering at some rate and exiting at some rate, then

"The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT."

These problems typically look like:

V	= volume of the vat	(liters)
t	= time	(min)
$y(t)$	= amount in vat	(kg)
$\frac{dy}{dt}$	= rate	(kg/min)

Thus,

$$\frac{dy}{dt} = \text{Rate In } - \text{Rate out}$$

$$= \left(?\ \frac{kg}{L}\right)\left(?\ \frac{L}{min}\right) - \left(\frac{y}{V}\ \frac{kg}{L}\right)\left(?\frac{L}{min}\right)$$

$$y(0) = ?\ \text{kg}$$

Example:

Assume a 100 Liter vat contains 5kg of salt initially. Two pipes (A & B) pump in salt water (brine).

Pipe A: Enters at 3L/min with a concentration of 4kg/L of salt.

Pipe B: Enters at 5L/min with a concentration of 2kg/L of salt.

The vat is well mixed.

The mixture leaves the vat at 8L/min.

Let $y(t)$ = the amount of salt in the vat at time t.

(a) Find $y(t)$.

(b) Find the limit of $y(t)$ as $n \to \infty$.

4. Air Resistance:

A skydiver steps out of a plane that is 4,000 meters high with and initial downward velocity of 0 m/s. The skydiver has a mass of 60 kg.

Let y(t) = "height at time *t*"
Let v(t) = y'(t) = "velocity at time *t*"
Let a(t)=v'(t)=''(t) = "accel. at time *t*"

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (and it is acting downward):

$F_g = -mg = -60 \cdot 9.8 = -588$ N

One model for air resistance

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k\,v \quad \text{Newtons}$$

Assume for this problem k = 12.

The Logistics Equation

Consider a population scenario where there is a limit to the amount of growth (spread of a rumor, for example).

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) \text{ with } P(0) = P_0$$

Let $P(t)$ = population size at time t.

M = maximum population size. (capacity)

We want a model that

…is like natural growth when P(t) is significantly smaller than M;

…levels off (with a slope approaching zero), then the population approaches M.

One such model is the so-called logistics equation

Random “scary-looking” problems

Spring 2011 Final:

Brief summary of what it says:

$v(t)$ = velocity of an object

$$F = mg - kv$$

Recall:

$$F = ma = m\frac{dv}{dt}$$

You are given m, g, and k and asked for solve for $v(t)$.

Spring 2014:

A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day.

Winter 2011

Your friend wins the lottery, and gives you P_0 dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously (a pretty good approximation to reality if you make regular frequent withdrawals) at a rate of \$3600 per year.

Fall 2009

The swine flu epidemic has been modeled by the Gompertz function, which is a solution of

$$\frac{dy}{dt} = 1.2\, y\, (K - ln(y)),$$

where $y(t)$ is the number of individuals (in thousands) in a large city that have been infected by time t, and K is a constant.

Time t is measured in months, with t = 0 on July 9, 2009.

On July 9, 2009, 75 thousand individuals had been infected.

One month later, 190 thousand individuals had been infected.